**Report on the thesis**
**ALGORITHMS FOR SYNCHRONIZING AUTOMATA**
**by Marek Szykuła**

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The thesis under review is devoted to a comprehensive theoretical and experimental study of synchronizing automata. An emphasis is made on algorithmic issues and questions related to the famous Černý conjecture.

The thesis consists of seven chapters. Chapter 1 gives a quick introduction in the thesis containing a historical overview and a brief outline of results. The outline creates an adequate impression of the author's goals and achievements. The historical overview is basically accurate but when discussing the origins of the concept of a synchronizing automaton in Section 1.1, the author should have mentioned coding theory in which the same notion had been discovered independently and simultaneously. I mean, in particular, the paper "Test for synchronizability of finite automata and variable length codes" by Shimon Even (IEEE Trans. Inform. Theory 10: 185–189, 1964), from which the name *synchronizing* seems to have originated. Also, when discussing synchronization of random automata in Subsection 1.3.3, the author fell yet another victim to a binding error in Volume 37 of "Semigroup Forum" where the paper '"The range order of a product of $i$ transformations from a finite full transformation semigroup" by Peter Higgins (Semigroup Forum 37:31–36, 1988) was published. Namely, the content of pages 35 and 36 of this volume had been swapped, and this creates the impression that the result formulated at the beginning of page 35 is a consequence of calculations performed in pages 33–34, and thus, this is an original result proved by Higgins. In fact, as one can read on page 36, this result is nothing but a well-known fact from mathematical genetics which arises out of the so-called Wright–Fisher model for the random selection of the parents of a given gene in a population of order $n$. Therefore, the claim that a random automaton with $n$ states and $2n$ letters is synchronizing with probability that tends to 1 as $n \to \infty$ (which is an automata-theoretic reformulation of the latter fact) should not be attributed to Higgins. There are some further minor inaccuracies, for instance, in page 2 the author first says that Černý formulated his conjecture in 1969 and then adds that in this year we celebrate the 50th anniversary of the conjecture. These little bugs and typos (such as 'there exist a synchronizing word' in page 1, line $-5$) are, of course, harmless.

Short Chapter 2 contains preliminaries sufficient to make the thesis to a reasonable extent self-contained. Here one can observe the author defines in page 19 only complete nondeterministic automata but in page 20 he introduces the inverse automaton of a deterministic automaton, and the inverse automaton need not be complete. Perhaps, the word 'complete' in the definition in page 19 is just a copy-pasting artefact that the author forgot to remove.

Chapters 3–7 contain the author's original contributions. Chapters 3, 4, and 6 present some advanced algorithms; Chapter 5 collects some theoretical results; and Chapter 7 adds a few important observations that have been obtained by theoretical analysis of outcomes of the author's computational experiments. Altogether the material of these five chapters constitutes a substantial achievement in the theory of synchronizing automata. The presentation is well organized and very polished, and this is not a surprise because these chapters are mostly based on published conference and journal papers.

Chapter 3 presents a new exact algorithm for computing reset words of minimum length for a given synchronizing automaton, a detailed account of the implementation of the algorithm, a complexity analysis, and an overview of experimental results. While the idea of the algorithm is rather straightforward, its efficient implementation has required several clever tricks and shows the author's skill as a highly qualified algorithmist. Experimental results convincingly demonstrate that the algorithm is by far superior over all previously known algorithms in the area.

Chapter 4 presents a new polynomial heuristic algorithm for finding short reset words. The chapter has a similar structure to the previous one.

Chapter 5 is devoted to upper bounds for reset length in some classes of synchronizing automata. The results of this chapter are mostly of incremental nature, exploring some known ideas and providing only slight improvements over previously existing bounds. Still, these improvements are worth registering: since the Černý conjecture has proved to be very hard, even a partial progress is of interest. There is a TEXnical problem in this chapter: it appears that the command `\eqref` has been redefined such that `\eqref{label}` produces "Equation (..)" rather than "(..)". This results in strange constructs such as "the Steinberg's bound Equation (5.1)" in page 66—this particular reference is especially confusing since (5.1) is an expression that involves no equality sign so that referring to it as "Equation" looks very weird indeed. I feel I should also mention some typos that occur in this chapter. In page 69, line −12, the expression $x_m - 1$ should be $x^m - 1$; in page 71, line −7, one should change $t$ in the expression $Q(ab)^t$ to some other letter since $t$ has been "frozen" in the formulation of Proposition 5.3.1. As already mentioned, in general the text is very polished and typos are rare and mostly harmless—these two is actually the only bugs that might cause some slight confusion. Also the phrase "It has no two the same consecutive letters" in the proof of Proposition 5.3.1 does not seem to be a good English phrase. Perhaps, this is a proper place to say that the overall level of the author's English is fairly good—there are some minor errors but they cause no or almost no difficulties in understanding the thesis.

Chapter 6 presents a recursive algorithm for generating non-isomorphic automata. Even though the algorithm is not immediately related to synchronization issues, I evaluate it as a serious achievement and also the most original part of the thesis. It is this algorithm that allows the author to perform exhaustive search through all ternary automata with up to 8 states—a task that was out of reach for all earlier approaches. I am sure that the algorithm will find many further applications in experiments with automata.

Chapter 7 contains a number of results of fundamental value. In particular, the example in Section 7.1 showing that a synchronizing automaton with $2m+1$ states may contain an $m$-element subset such that its shortest extending word is of length $> \frac{m^2-1}{2}$ is very surprising and solves (in the negative) an important conjecture. The author also formulates several interesting conjectures that may in my opinion serve as reasonable guidelines for further research in the area.

Summarizing I can say that the quantity, the quality, and the relevance of the presented research to current trends in the area of synchronizing automata is certainly enough to make it a very good doctoral thesis. The author has demonstrated strong research potential and deep knowledge in several crucial directions of automata theory. I therefore recommend that the dissertation is accepted and that the PhD defense of Marek Szykuła proceeds to its next phase.

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